Renormalized scalar propagator around a dispiration

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The renormalized Feynman propagator for a scalar field in the background of a cosmic dispiration (a disclination plus a screw dislocation) is derived, opening a window to investigate vacuum polarization effects around a cosmic string with dislocation, as well as in the bulk of an elastic solid carrying a dispiration. The use of the propagator is illustrated by computing vacuum fluctuations. In particular it is shown that the dispiration polarizes the vacuum giving rise to an energy momentum tensor which, as seen from a local inertial frame, presents nonvanishing off-diagonal components. Such a new effect resembles that where an induced vacuum current arises around a needle solenoid carrying a magnetic flux (the Aharonov-Bohm effect), and may have physical consequences. Connections with a closely related background, namely the spacetime of a spinning cosmic string, are briefly addressed.

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I. INTRODUCTION

Computations on quantum fields in locally flat backgrounds with linear defects not only are an interesting matter of principle but may also be useful in physical contexts as different as cosmology (the gravitational background of a cosmic string [1]) and condensed matter physics (the effective geometry associated with linear defects in solids [2,3]). The nontrivial global geometry of the locally flat background around a linear defect induces vacuum polarization [4–6]. Casimir-like effects near cosmic strings and in the bulk of a solid carrying a linear defect have been investigated throughout the years in a variety of situations (see e.g. [7–11]). The literature addresses mainly conical defects appearing in connection with the geometrical background of an ordinary cosmic string [1], and of a disclination in a continuous elastic solid [2,3].

This work begins an investigation of quantum field theory effects in a spacetime with a cosmic dispiration. Classical fields were recently considered in Ref. [12], and investigations on quantum theory in related backgrounds have been carried out in Refs. [13–16]. In Ref. [17], it has been conjectured that the geometry associated with a cosmic dispiration may correspond to the gravitational field of certain chiral cosmic strings [18].

The geometry that arises in the context of general relativity [17] as well as in the Einstein-Cartan theory [19–21] is presented in the next section. In Sec. III, the proper time expression (see e.g. [22]) of the Feynman propagator for a scalar field $\phi(x)$ in the four-dimensional spacetime of a cosmic dispiration is obtained. Additional manipulations yield a propagator which is renormalized with respect to the Minkowski vacuum. This renormalized propagator is then used in Sec. IV to evaluate the vacuum fluctuations $\langle \phi^2\rangle$ and $\langle T^{\mu}{}_{\nu}\rangle$ in the limit of small masses. Section IV also contains a detailed analysis of the behavior of $\langle \phi^2\rangle$ where disclination and screw dislocation effects are confronted. Final remarks are outlined in Sec. V, including a short discussion on global hyperbolicity in the context of a related background.

Throughout the text $c=\hbar=1$.

II. THE GEOMETRY

The geometry of the spacetime of a cosmic dispiration (a cosmic string with dislocation [17]) is characterized by the Minkowski line element written in cylindrical coordinates [17,19],

$$ds^2=dt^2-dr^2-r^2d\varphi^2-dZ^2, \tag{1}$$

and by the identification

$$(t,r,\varphi,Z)\sim(t,r,\varphi+2\pi\alpha,Z+2\pi\kappa), \tag{2}$$

where α and κ are parameters corresponding to a disclination and a screw dislocation, respectively. By defining new space coordinates $\theta:=\varphi/\alpha$ and $z:=Z-(\kappa/\alpha)\varphi$, Eq. (1) becomes $ds^2=dt^2-dr^2-\alpha^2r^2d\theta^2-(dz+\kappa d\theta)^2$, and the usual identification $(t,r,\theta,z)\sim(t,r,\theta+2\pi,z)$ must be observed. The case for which $\alpha=1$ and $\kappa=0$ clearly corresponds to the Minkowski spacetime.

In the context of general relativity Eq. (2) encodes the fact that Eq. (1) hides a curvature singularity on the symmetry axis. (In the Einstein-Cartan theory there exists also a torsion singularity on the symmetry axis, when $\kappa\neq 0$.)

III. THE RENORMALIZED PROPAGATOR

The Feynman propagator for a scalar field with mass m in the background described above is a solution of [4]

$$(\Box_x+m^2)G_{\mathcal{F}}(x,x')=-\frac{1}{r}\,\delta(x-x'), \tag{3}$$

where $\Box_x$ is just the d'Alembertian in Minkowski spacetime written in cylindrical coordinates. By taking into account Eq. (2), the non-trivial geometry manifests itself only through

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$$G_{\mathcal{F}}(\varphi+2\pi\alpha,Z+2\pi\kappa)=G_{\mathcal{F}}(\varphi,Z), \tag{4}$$

where the other coordinates were omitted.

The eigenfunctions of the operator $\Box_x+m^2$, which are regular at $r=0$ and satisfy Eq. (4), are given by

$$\psi_{\omega,\mu,n,\nu}(x)=\frac{1}{(2\pi)^{3/2}\alpha^{1/2}}J_{|n-\nu\kappa|/\alpha}(\mu r)e^{i[\nu Z-\omega t+(n-\nu\kappa)\varphi/\alpha]}, \tag{5}$$

where n is an integer, ω and ν are real numbers, μ is a positive real number and J_σ denotes a Bessel function of the first kind. The corresponding eigenvalues are $E_{\omega,\mu,\nu}=\mu^2+\nu^2-\omega^2+m^2$, and a convenient (but rather unusual) normalization has been used [23].

Using the Bessel functions completeness relation [24]

$$\int_0^\infty dk\ kJ_\sigma(kr)J_\sigma(kr')=\frac{1}{r}\delta(r-r'),$$

and the Fourier expansion (Poisson's formula)

$$\sum_{n=-\infty}^{\infty}\delta(\theta+2\pi n)=\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}e^{in\theta}, \tag{6}$$

direct application of $\Box_x+m^2$ shows that [23]

$$G_{\mathcal{F}}(x,x')=-i\int_0^\infty dT\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}d\nu\int_0^\infty d\mu \times\mu e^{-iTE_{\omega,\mu,\nu}}\psi_{\omega,\mu,n,\nu}(x)\psi^*_{\omega,\mu,n,\nu}(x'), \tag{7}$$

where the integration over T is regularized by subtracting an infinitesimal imaginary term from $E_{\omega,\mu,\nu}$. Evaluating the integrations over ω and μ [25], Eq. (7) yields

$$G_{\mathcal{F}}(x,x')=-\frac{(i\pi)^{1/2}}{16\pi^3\alpha}\int_0^\infty\frac{dT}{T^{3/2}}e^{i\{[r^2+r'^2-(t-t')^2]/4T-m^2T\}}\times\int_{-\infty}^{\infty}d\nu e^{-iT\nu^2+i[(Z-Z')-\kappa(\varphi-\varphi')/\alpha]\nu}\times\sum_{n=-\infty}^{\infty}I_{|n-\nu\kappa|/\alpha}(rr'/2iT)e^{in(\varphi-\varphi')/\alpha}, \tag{8}$$

where I_σ denotes a modified Bessel function of the first kind.

In order to tackle renormalization, a convenient integral representation for I_σ [25] is used to obtain the equality,

$$\sum_{n=-\infty}^{\infty}I_{|n+\beta|/\alpha}(y)e^{in\delta/\alpha}=\alpha e^{y\cos\delta-i\beta\delta/\alpha}-\frac{1}{\pi}\int_0^\infty d\tau\ \ e^{-y\cosh\tau}\times\sum_{n=-\infty}^{\infty}\sin(|n+\beta|\pi/\alpha)e^{-(|n+\beta|\tau-in\delta)/\alpha}, \tag{9}$$

which holds for $\alpha>1/2$ (smaller values can be considered by taking into account terms that were omitted), and is crucial to implementing renormalization and to performing the integration over ν in Eq. (8). (It should be pointed out that the integral representation for I_σ mentioned above has previously been used in related contexts [14,26].)

Using Eq. (9) in Eq. (8), we obtain

$$G_{\mathcal{F}}(x,x')=G^0_{\mathcal{F}}(x,x')+G^{(\alpha,\kappa)}(x,x'), \tag{10}$$

where $G^0_{\mathcal{F}}(x,x')$ is the Feynman propagator in the Minkowski spacetime and

$$G^{(\alpha,\kappa)}(x,x'):=\frac{(i\pi)^{1/2}}{16\pi^4\alpha}\int_0^\infty\frac{dT}{T^{3/2}}\ e^{i\{[r^2+r'^2-(t-t')^2]/4T-m^2T\}}\times\int_{-\infty}^{\infty}d\nu e^{-iT\nu^2+i[(Z-Z')-\kappa(\varphi-\varphi')/\alpha]\nu}\times\sum_{n=-\infty}^{\infty}e^{-in(\varphi-\varphi')/\alpha}\sin(|n+\nu\kappa|\pi/\alpha)\times\int_0^\infty d\tau e^{-(rr'/2iT)\cosh\tau-|n+\nu\kappa|\tau/\alpha}. \tag{11}$$

Observing the sine factor in Eq. (11), for $\alpha=1$ and $\kappa=0$ Eq. (11) vanishes, leaving in Eq. (10) only the Minkowski contribution, as should be. In order to build up observables with $G_{\mathcal{F}}(x,x')$ (cf. the next section) its ultraviolet divergences must be eliminated, which can be done simply by dropping $G^0_{\mathcal{F}}(x,x')$ in Eq. (10) (as the background geometry of a cosmic dispiration is locally flat [cf. Eq. (1)], all the ultraviolet divergences are encapsulated in the Minkowski contribution [6]).

A more workable expression for the renormalized propagator of a massless scalar field $D^{(\alpha,\kappa)}(x,x')$ can be obtained by inserting in Eq. (11) $\delta[\lambda-(n+\nu\kappa)/\alpha]$ (obviously accompanied by integration over λ). Recalling that $f(x)\delta(x-y)=f(y)\delta(x-y)$, one uses Eq. (6) before evaluating the Gaussian integration over ν. Finally, the integrations over λ and T are separately evaluated [25], resulting in

$$D^{(\alpha,\kappa)}(x,x')=\frac{i}{2\pi^2}\sum_{n=-\infty}^{\infty}\int_0^\infty d\tau\frac{[\tau^2+\pi^2-(2\pi\alpha n-\Delta\varphi)^2][r^2+r'^2+2rr'\cosh\tau+(\Delta Z-2\pi n\kappa)^2-(\Delta t)^2]^{-1}}{\{[\pi(2\alpha n+1)-\Delta\varphi]^2+\tau^2\}\{[\pi(2\alpha n-1)-\Delta\varphi]^2+\tau^2\}}, \tag{12}$$

where $\Delta t := t - t'$, and likewise for φ and Z.

As $\kappa \to 0$, the summation in Eq. (12) can be evaluated by considering the power series expansion of $\psi(x)$ (the logarithmic derivative of the gamma function) and its properties, yielding as the leading contribution a familiar integral representation [27] for the renormalized scalar propagator in an ordinary conical background ($\kappa = 0$). On the other hand, as $\kappa \to \infty$, the expression for the leading behavior is obtained from Eq. (12) by ignoring the summation and by setting $n = 0$, resulting in an integral representation which does not depend on either the parameters characterizing the cosmic dispiration.

IV. APPLICATION

In the following Eq. (12) will be used to calculate vacuum averages.

A. $\langle \phi^2(x) \rangle$

Formally (see e.g. [4]), the vacuum fluctuation $\langle \phi^2(x) \rangle$ can be obtained by setting $x' = x$ in Eq. (12), and multiplying the resulting expression by i,

$$\langle \phi^2(r) \rangle = -\frac{1}{8\pi^2 r^2}\int_0^\infty d\tau \frac{1}{(\pi^2+\tau^2)\cosh^2(\tau/2)} - \frac{1}{4\pi^2 r^2}\int_0^\infty d\tau \times \sum_{n=1}^{\infty} \frac{\tau^2 - \pi^2(4\alpha^2 n^2 - 1)}{[\pi^2(2\alpha n+1)^2+\tau^2][\pi^2(2\alpha n-1)^2+\tau^2][\cosh^2(\tau/2)+(n\pi\kappa/r)^2]}, \tag{13}$$

which does not depend on the direction of the screw dislocation, i.e., on the sign of κ. The considerations at the end of the previous section lead to the fact that as $\kappa/r \to 0$, Eq. (13) yields as the leading contribution

$$\langle \phi^2(r) \rangle = \frac{1}{48\pi^2 r^2}(\alpha^{-2} - 1), \tag{14}$$

which is the known [28] behavior around an ordinary cosmic string. As $\kappa/r \to \infty$, the leading contribution is now given by

$$\langle \phi^2(r) \rangle = -\frac{1}{48\pi^2 r^2}, \tag{15}$$

where the integral in the first term of the right hand side of Eq. (13) was evaluated numerically. [It is rather curious that Eq. (15) follows from Eq. (14) by setting $\alpha \to \infty$.]

Apart from theses asymptotic behaviors, the dependence of $\langle \phi^2(r) \rangle$ on r is nontrivially hidden in Eq. (13), requiring numerical analysis. The plots (where units are omitted) show how $\langle \phi^2(r) \rangle$ varies with r for various combinations of values of α and κ.

When $\kappa \neq 0$, Eq. (14) shows that for very large values of r disclination effects are dominant (see Figs. 1 and 2), whereas for very small values of r, according to Eq. (15), screw dislocation effects rule in a way typical of vacuum fluctuations near boundaries [5] [near the dispiration $\langle \phi^2(r) \rangle$ is essentially independent of the cosmic dispiration attributes].

For arbitrary values of r, numerical and analytical examination shows that when $\alpha \geqslant 1$, $\langle \phi^2(r) \rangle \leqslant 0$ (see Figs. 2 and 3). When $\alpha < 1$ the screw dislocation and disclination effects compete, causing $\langle \phi^2(r) \rangle$ to have a maximum which, by simple dimensional considerations, is found to be proportional to $1/\kappa^2$, and whose corresponding $r = r_M$ is propor-

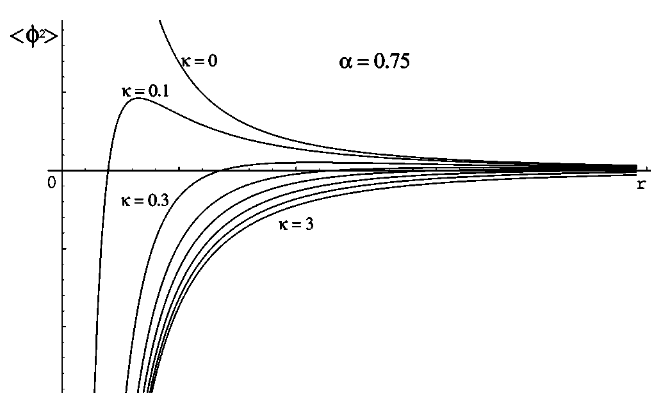

FIG. 1. The intermediate plots, from the bottom, correspond to $\kappa = 1.5, 1, 0.7, 0.5$, respectively.

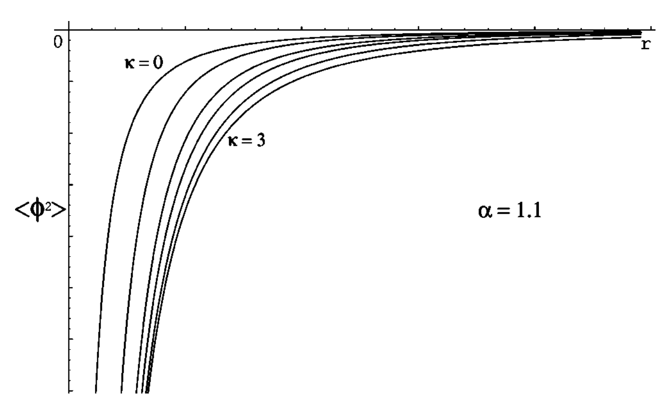

FIG. 2. The intermediate plots, from the bottom, correspond to $\kappa = 1, 0.5, 0.3, 0.1$, respectively.

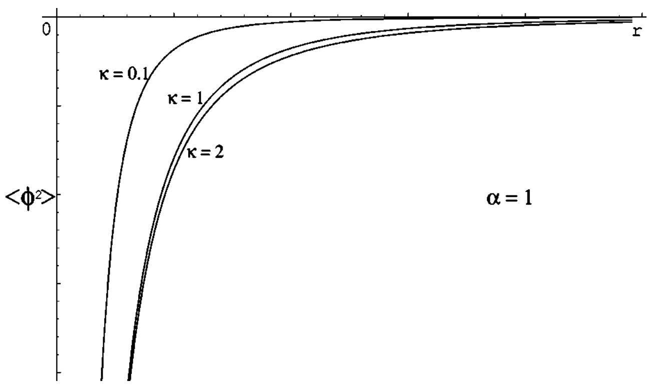

FIG. 3. Plots corresponding to screw dislocation effects.

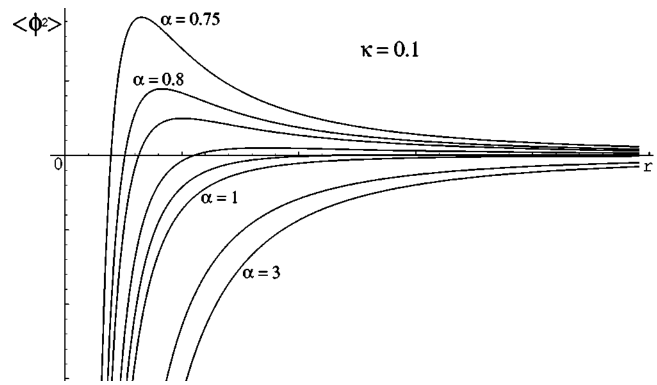

FIG. 4. The intermediate plots, from the bottom, correspond to $\alpha=1.5,0.95,0.9,0.83$, respectively.

tional to $|\kappa|$ (see Figs. 1 and 4). A similar analysis reveals that, when $\alpha<1$, $\langle\phi^2(r)\rangle$ vanishes at $r=r_0$, which is obviously also proportional to $|\kappa|$.

One notices that, for any $\alpha<1$, r_0 and r_M move toward $r=0$ as $|\kappa|$ decreases (see Fig. 1). For a given $\kappa\neq0$, as α decreases from unity, r_0 and r_M also move toward $r=0$ (see Fig. 4).

It should be appreciated that when $\alpha<1$, the $\kappa=0$ and $\kappa\neq0$ behaviors differ radically from each other as $r\to0$; namely, for $\kappa=0$ $\langle\phi^2(r)\rangle$ diverges positively, whereas for $\kappa\neq0$ $\langle\phi^2(r)\rangle$ diverges negatively [see Eq. (14), Eq. (15) and Fig. 1].

It should be pointed out that, when $\kappa\neq0$, Eq. (14) is the leading contribution as $r\to\infty$ only when $\alpha\neq1$. When $\alpha=1$, Eq. (14) vanishes and the sub-leading contribution, due to the screw dislocation, takes over. Dimensional considerations may suggest that such a contribution is proportional to κ^2/r^4. Nevertheless, an analysis seems to show that $\langle\phi^2(r)\rangle$ falls differently as $r\to\infty$ (see Figs. 3 and 4). [In fact, Eq. (13) is not very handy to determine sub-leading contributions, and an alternative expression can be more useful for this purpose.]

B. $\langle T^\mu{}_\nu(x)\rangle$

As is well known (see e.g. Ref. [4]) the vacuum expectation value of the energy momentum tensor can formally be obtained by applying the differential operator

$$\mathcal{D}^\mu{}_\nu(x,x'):=(1-2\xi)\nabla^\mu\nabla_{\nu'}-2\xi\nabla^\mu\nabla_\nu+(2\xi-1/2)\delta^\mu{}_\nu\nabla^\lambda\nabla_{\lambda'} \tag{16}$$

to the renormalized scalar propagator,

$$\langle T^\mu{}_\nu\rangle=i\lim_{x'\to x}\mathcal{D}^\mu{}_\nu(x,x')D^{(\alpha,\kappa)}(x,x'). \tag{17}$$

By using Eqs. (12) and (17), a detailed analytical and numerical study of $\langle T^\mu{}_\nu\rangle$ (along the lines of that presented above for $\langle\phi^2\rangle$) is possible, but rather lengthy to be included here. Only the behavior of $\langle T^\mu{}_\nu\rangle$ far away from the dispiration will be presented below.

As mentioned previously, when $\kappa\to0$ the dominant contribution in Eq. (12) is the renormalized propagator around a disclination. It follows that as $\kappa/r\to0$, Eq. (17) yields for the diagonal components essentially the expressions long known in the literature for the vacuum fluctuations around an ordinary cosmic string ($\kappa=0$) [7–9]. Regarding the remaining components, the prescription in Eq. (17) kills off the dominant contribution in Eq. (12), with the result that the subleading contribution yields two nonvanishing off-diagonal components,

$$\langle T^\varphi{}_Z\rangle=\frac{i}{r^2}\lim_{x'\to x}\partial_\varphi\partial_Z D^{(\alpha,\kappa)}(x,x')=\frac{\kappa}{r^6}B(\alpha) \tag{18}$$

and

$$\langle T^Z{}_\varphi\rangle=\frac{\kappa}{r^4}B(\alpha), \tag{19}$$

where

$$B(\alpha):=\frac{1}{32\pi^3\alpha^2}\int_0^\infty d\tau\frac{\alpha\sin(\pi/\alpha)[\cos(\pi/\alpha)-\cosh(\tau)+\tau\sinh(\tau)]-\pi[\cos(\pi/\alpha)\cosh(\tau)-1]}{[\cosh(\tau)-\cos(\pi/\alpha)]^2\cosh^4(\alpha\tau/2)}. \tag{20}$$

It is worth remarking that, unlike the diagonal components, $\langle T^\varphi{}_Z\rangle$ and $\langle T^Z{}_\varphi\rangle$ do not depend on the coupling parameter ξ.

The plot of $B(\alpha)$ against the disclination parameter α is shown in Fig. 5. When $\alpha=1$, the integration in Eq. (20) can be analytically evaluated [25], resulting in $B=1/60\pi^2$ which corresponds approximately to the value of α suggested by the physics of formation of ordinary cosmic strings [1].

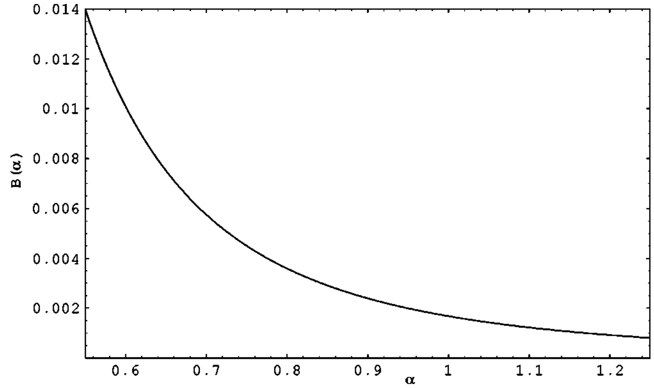

FIG. 5. Plot of $B(\alpha)$ versus α.

It is instructive to display both disclination and screw dislocation effects in the same array. When $\xi = 1/6$ (conformal coupling), for example, $\langle T^{\mu}{}_{\nu} \rangle$ with respect to the local inertial frame [see Eq. (1)] can be cast into the form

$$\langle T^{\mu}{}_{\nu} \rangle = \frac{1}{r^4} \begin{pmatrix} -A & 0 & 0 & 0 \\ 0 & -A & 0 & 0 \\ 0 & 0 & 3A & \kappa B/r^2 \\ 0 & 0 & \kappa B & -A \end{pmatrix}, \tag{21}$$

where $A(\alpha) := (\alpha^{-4} - 1)/1440\pi^2$, and which holds far away from the defect (and for $\alpha \neq 1$, when $\kappa \neq 0$). [When $\kappa \neq 0$, by setting $\alpha = 1$ in Eq. (21), A vanishes and subleading contributions depending on κ take over.]

V. FINAL REMARKS

This work presented a renormalized expression for the Feynman propagator of a scalar field $\phi(x)$ around a cosmic dispiration. The propagator was then used to calculate $\langle \phi^2(x) \rangle$ and the asymptotic behavior of $\langle T^{\mu}{}_{\nu}(x) \rangle$ as preliminary exercises to tackle more elaborate vacuum fluctuations. The expectation is that the results may be useful in cosmology and condensed matter physics. A few remarks are in order.

It was argued in the previous section that, as far as $\langle \phi^2(r) \rangle$ is concerned, disclination effects are dominant over screw dislocation effects when $r \to \infty$, and the other way around when $r \to 0$. This result should not be extended to all vacuum fluctuations without caution, since some of them are obtained from the renormalized propagator by applying prescriptions which may eliminate the dominant contribution in Eq. (12) [see e.g. Eqs. (18) and (19)].

The eigenfunctions in Eq. (5) have the form

$$R(r)\chi(\varphi)\exp\{i(\nu Z - \omega t)\},$$

where $\chi(\varphi + 2\pi\alpha) = \exp\{-i2\pi\nu\kappa\}\chi(\varphi)$. This boundary condition is typical of the Aharonov-Bohm setup where $\nu\kappa$ is identified with the flux parameter $e\Phi/2\pi$. By carrying over to the four-dimensional context lessons from gravity in three dimensions [29,30], it follows that the charge e and the magnetic flux Φ should be identified with the longitudinal linear momentum ν and $2\pi\kappa$, respectively [17]. (In fact the roots of this analogy lie in the gauge theory aspects of gravity.) According to this picture, one interprets the polarization effect displayed in Eq. (19) in the following manner. A dispiration (more precisely, a screw dislocation) polarizes the vacuum of a scalar field, inducing a flux of longitudinal linear momentum around the defect. Such a flux depends on the direction of the screw dislocation (i.e., on the sign of κ) in the same way that vacuum currents around a needle solenoid depend on the direction of the magnetic flux [31].

Symmetry considerations show that the geometrical background of a cosmic dispiration is closely related to that of a spinning cosmic string [17], which is also locally flat [see Eq. (1)] with a time helical structure characterized by the identification $(t,r,\varphi,Z) \sim (t + 2\pi S, r, \varphi + 2\pi\alpha, Z)$ [instead of the space helical structure in Eq. (2)]. Such a helical time structure poses serious difficulties when implementing quantization, since the corresponding spacetime is not globally hyperbolic [5]. In fact, by insisting on implementing quantization with the usual procedures, one ends up with observables presenting pathological behavior [15,16,29]. [$\langle \phi^2(r) \rangle$ around a spinning cosmic string can be obtained from Eq. (13) by taking $\kappa \to iS$, rendering the vacuum fluctuation divergent [15].]

The summation in Eq. (13) could, in fact, be evaluated. The resulting integral representation for $\langle \phi^2(r) \rangle$, however, does not offer either analytical or computational advantage. It is given by Eq. (9) of Ref. [15], after replacing S by $i\kappa$. [Incidentally, it can be noticed that the factor $\sin(x/\alpha)$ in Eq. (9) of Ref. [15] is a misprint which should be read as $\sinh(x/\alpha)$.]

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